An algebraic max-plus model for HVLV systems scheduling and optimization with repetitive and flexible periodic preventive maintenance: just-in-time production

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9^{th} *International Conference of Modeling, Optimization and Simulation - MOSIM'12*
June 6-8, 2012 - Bordeaux - France
"Performance, interoperability and safety for sustainable development"

AN ALGEBRAIC MAX-PLUS MODEL FOR HVLV SYSTEMS SCHEDULING AND OPTIMIZATION WITH REPETITIVE AND FLEXIBLE PERIODIC PREVENTIVE MAINTENANCE: JUST-IN-TIME PRODUCTION

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ABSTRACT: *Most production scheduling problems, including High-Variety, Low-Volume (HVLV) scheduling problems assume that machines are continuously available. However, in most actual situations, machines become unavailable during certain periods when preventive maintenance (PM) is scheduled. In this paper, a HVLV scheduling problem is proposed while considering machines availability constraints. Each machine is subject to PM while maintaining flexibility in the start time of maintenance activities during the planning period. In this paper, two situations are investigated. First, the maintenance tasks are periodically scheduled: maintenance is required after a periodic time (all periods are equals on each machine). Second, time intervals between two consecutive maintenance activities are not equals (flexible periodic maintenance). However, time intervals are known in advance. Consequently, the maintenance operations are controllable. The jobs and the maintenance activities are scheduled simultaneously. Also, the maintenance tasks are scheduled between them, such that a regular criterion is optimized. In order to illustrate the performance of the proposed methodology, a simulation example is given.*

KEYWORDS: *HVLV manufacturing systems, max-plus scheduling and control model, decision variables, non-linear optimization, preventive maintenance, makespan and total tardiness, JIT production.*

1 INTRODUCTION

Due to their importance both in the fields of manufacturing industries and operations research, production scheduling and maintenance planning have been received considerable attention both in academia and in industry. Indeed, production plans and maintenance activities are two major issues in manufacturing industries. Production scheduling deals with finding the appropriate assignment of jobs on machines in order to obtain especial objectives by considering the existing constrains.

Numerous prior studies are dedicated to solve these problems in different workshops (single machine, parallel machines, flow-shop, job-shop and HVLV systems). Our study deals with High-Variety, Low-Volume (HVLV) manufacturing systems which are a class of dynamic systems where the behaviour can be assimilated to Discrete Event Dynamic Systems (DEDS). They are characterized by a wide variety of products using shared machines, a weak and personalized demand, relatively long processing times and frequent change over and set-up times. Consequently, a continuous approximation of the production flow by continuous flow systems (Tamani et al., 2009), (Tamani et al., 2011) and (Tamani et al., 2011a) is not appropriate for HVLV systems. In this framework, it seems very interesting to handle this kind of systems as Job-Shop systems (Huang and Irani, 2003) due to the wide variety of processed products.

Most of the production scheduling problems are NP-hard (Garey and Johnson, 1979), especially HVLV systems scheduling problems. In the production scheduling point of view, a significant part of literature assumes that machines are always available during the planning time horizon. However, in actual manufacturing systems, this assumption is unreasonable because, some unavailability periods, like maintenance activities, cause the machines to be not available for processing (Schmidt, 2000). For tackling this problem, several researchers have included recently unavailability periods like maintenance activities in their studies (Zribi et al., 2008) and (Sbihi and Vernier, 2008).

In this framework, to deal with sequencing decisions, control variables have been introduced in the scheduling model (Nasri et al., 2011). A dioid algebraic model has been developed to generate all feasible schedules by choosing different values for decision variables. This model is non-linear in the sense of (max, +) algebra. Moreover, in the case of synchronized systems, the proposed (max, +) models in (Nasri et al., 2011a) and (Nasri et al., 2011b) are linear.

Our contribution is to propose an analytical formulation of a dynamic scheduling for HVLV systems while Preventive Maintenance (PM) is considered using the (max, +) algebra. In this context, PM is integrated into the proposed model using (max, +) algebra mathematical relations. Compared to (Sbihi and Varnier, 2008) where the authors consider equal durations of maintenance tasks in the case of single machine scheduling, in this paper the allocated times to maintenance operations can be not equals on each machine. Indeed, the (max, +) model is a simple representation where only sequencing type decisions are needed to solve the conflicts between concurrent operations. The operations sequencing and maintenance activities are determined by incorporating decision variables in the model. In addition, different kinds of maintenance operations are scheduled via control variables, so that a regular criterion is optimized. In this context, the makespan is firstly minimized and then the total tardiness subject to a Just-In-Time (JIT) production is optimized . Two kinds of maintenance tasks are incorporated to the proposed model: repetitive periodic maintenance operations with equal periods on each machine and flexible periodic maintenance activities with different time intervals between two consecutive maintenance tasks.

The remainder of this paper is organized as follows: Section 2 gives a short review for the state-space HVLV systems scheduling modeling (Nasri et al., 2011). In Section 3, PM is considered. Next, an illustrative example of a (6x6) Job-Shop HVLV system with PM according to a non-linear optimization procedure is presented. Concluding remarks and future research directions are presented in Section 5.

2 HVLV SYSTEMS SCHEDULING MODELING WITHOUT PREVENTIVE MAINTENANCE

The focus of this section concerns a review for the HVLV systems scheduling using (max, +) algebra. For more details, readers are invited to read (Nasri et al., 2011).

2.1 Approach Principle

(Max, +) algebra is applied as a modeling tool in order to represent the scheduling problem of HVLV systems where relationships between the starting times of the operations require the maximum and addition operators. In order to generate feasible schedules on machines, the control variables used in the proposed model in the case of minimization of the makespan are the decision variables.

A dioid is considered as a set D with two operators, $\oplus$ and $\otimes$. The operation $\oplus$ called addition, produces in D a structure of a commutative monoid and has a neutral element ϵ called zero. The other operation, $\otimes$, called multiplication, produces in D a structure of a monoid and has a neutral element e, called unity. (Max, +) is a dioid, which consists of the real numbers R extended to include $-\infty$. (Max, +) algebra is used in development of algebraic models of DEDS (Baccelli et al., 1992). For all $a, b \in R \cup -\infty$ the max-plus operators are defined according to the following equations:

$$a \oplus b = max(a, b) \tag{1}$$

$$a \otimes b = a + b \tag{2}$$

2.2 Max-Plus Scheduling Model For HVLV Systems

Let us now firstly present the construction principle of the (max, +) algebraic model for the static (without maintenance) scheduling problem for the HVLV systems. The knowledge of the following informations are needed to establish our model:

- The individual operations and route for each job.
- The machines on which each operation should be executed.
- The predecessors of each operation (the process plan for each job).
- External starting conditions of each operation (the times at which raw materials are fed to the system and the starting date of a new scheduling in a new planning horizon).

The incorporation of the decision variables into the model is satisfied by the fact that the sequencing of operations for different products on the same machine requires a decision on the order in which the operations are processed such that the conflicts are resolved and precedence constraints are not violated. Moreover, to get feasible schedules, constraints are added in order to bound decision variables.

It was shown in (Nasri et al., 2011) that the developed event-timing-equations describing the dynamic of the system can be grouped into the following (max, +) matrix form:

$$X = T \oplus U \oplus C \otimes X \quad (3)$$

where:

- X is a (Nx1) (max, +) state vector (N is the total number of operations) which collects the starting times of operations.
- T is a (Nx1) (max, +) vector which is composed of the beginning dates of the scheduling over the new planning horizon.
- U is a (Nx1) vector which contains the different dates at which the raw material of each product is fed to the system.
- C is a (NxN) appropriate (max, +) matrix describing the relationships among different state variables of the system. It contains the different decision variables.

3 STATEMENT OF HVLV SYSTEMS SCHEDULING PROBLEM WITH MAINTENANCE USING (MAX, +) ALGEBRA

The HVLV system scheduling problem with maintenance activities that we addressed here can be described as follows:

A set of n jobs J={J_1, J_2,...,J_n} is to be processed on a set of m machines denoted by M={M_1, M_2,...,M_m}. Each job i consists of a sequence of n_j operations (routing). Each operation $O_{ijk}(1 \leqslant i \leqslant n, 1 \leqslant j \leqslant n_j, 1 \leqslant k \leqslant m)$ has to be performed to complete one job. We consider $h(h = 1, ..., x)$ periodic maintenance activities to be processed on each machine M_k during the planning horizon based on a predefined maintenance policy. Two PM cases are investigated:

- Repetitive periodic maintenance: the time intervals between two consecutive maintenance tasks are equals. Maintenance periods are periodically fixed: maintenance is required after a periodic time interval (e.g., periodical maintenance with m equal periods $T_k(k = 1, ..., m)$ on each machine M_k (Figure 1). Moreover, the durations allocated to the maintenance activities can be not equals. Note that in (Sbihi and Varnier, 2008) the durations of maintenance operations are equals on a single machine.
- Flexible periodic maintenance: the time intervals between two consecutive flexible periodic maintenance activities PM_{ik} and PM_{jk} , noted $\triangle_{ik,jk}$ $(i, j = 1...x, i \neq j)$ are not equals but fixed in advance. In addition, the durations allocated to maintenance activities can be not equals. Also, the starting date of the first flexible maintenance is considered known in advance (Figure 2).

The maintenance activities can be considered as operations of particular jobs. Consequently, they can be incorporated in the equation 28. Indeed, in both sit-

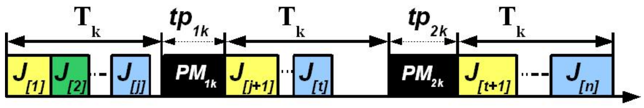

Figure 1: A schedule on a single machine with periodic maintenance: $J_{[i]}$ is the number of job in ith position and PM_{hk} is the hth operation of maintenance $(h = 1, ..., x)$ on machine M_k.

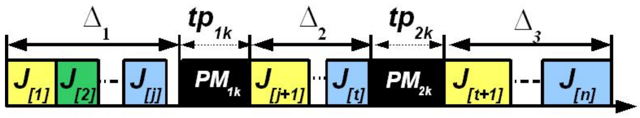

Figure 2: A schedule on a single machine with flexible periodic maintenance: $J_{[i]}$ is the number of job in ith position and PM_{hk} is the hth operation of maintenance $(h = 1, ..., x)$ on machine M_k.

uations, the maintenance tasks are controllable. Jobs and maintenance operations are scheduled simultaneously, so that a regular criterion is optimized.

$\triangle_{ik,jk}$ $(i, j = 1...x, i \neq j)$ represents the time interval between two consecutive flexible periodic maintenance operations: PM_{ik} and PM_{jk} on machine M_k. The time intervals are different but fixed in advance. Moreover, the starting date of the first flexible maintenance is considered known in advance.

Each Preventive Maintenance PM_{hk} has a deterministic duration denoted by tp_{hk} where k is the index of machine. Let xp_{hk} is the starting time of the maintenance PM_{hk} on machine M_k. Then the proposed dynamic (max, +) scheduling model has the starting times of operations and the starting times of maintenance activities as events of the system (Figure 3).

The proposed (max, +) model objective is to handle simultaneously scheduling production jobs and maintenance activities. Two kinds of maintenance are considered: repetitive periodic maintenance and flexible periodic maintenance. The scheduling of production jobs can be described by the same event-timing equations shown in (Nasri et al., 2011). Moreover, a new part is introduced into the model to represent the dynamic scheduling between maintenance activities and the scheduling between operations and maintenance activities. Two regular criteria are optimized:

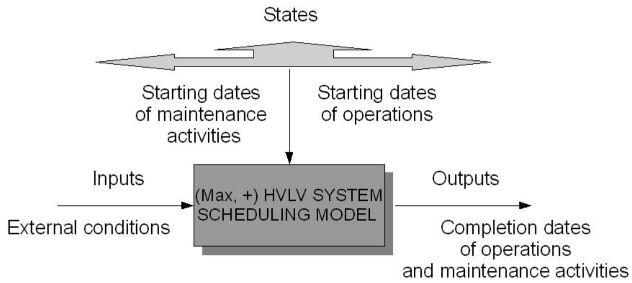

Figure 3: HVLV systems scheduling modeling principle integrating maintenance.

- The makespan is firstly minimized using a non-linear optimization with constraints in (max, +) algebra. In this case, decision variables are considered as the variables of optimization (the control variables of the system). They are determined, such that the proposed model generates all feasible schedules between operations and maintenance activities processed on the same machines.

- The total tardiness is then minimized using a non-linear optimization with constraints in (max, +) algebra. In this case, control variables are the decision variables that generate feasible schedules and the times at which raw materials of products are fed as late as possible to the system subject to a JIT production.

3.1 Max-Plus Scheduling Modeling Of HVLV Systems With Repetitive Periodic Maintenance Tasks

In this section, only repetitive periodic maintenance activities with equal periods on machines are considered. Consequently, maintenance periods are periodically fixed: maintenance is required, on each machine M_k, after a periodic time interval $T_k(k = 1, ...m)$. Within this context, a (max, +) HVLV model with periodic maintenance is proposed. In this model, both the starting time of operations and maintenance tasks are considered as the events (states) of the system. Then $\forall \ 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant n_j, 1 \leqslant k \leqslant m$ and $1 \leqslant h \leqslant x$, two situations can be distinguished:

If operation $j \in P$ is the first (i.e., unprecedented) operation on the job, then its processing start time x_{ijk} is determined by the maximum of either:

- The starting date t of the new scheduling over the new planning horizon.

- The date u_i at which the raw material of its corresponding product i is fed to the system.

- The completion of other operations ($j' \neq j$, and $j' \in P$), for other products i' that require processing on machine k. This is determined by the decision variables $V_{ijk,i'j'k}$ that determine which operation must be processed earlier on machine k.

- The completion of periodic maintenance activities PM_{hk} that require processing on machine M_k. This is determined by the decision variables $V_{ijk,hk}$ that generate the schedule between operation O_{ijk} and the periodic maintenance PM_{hk}.

This situation is formulated as follow:

$$x_{ijk} = max(\quad t; u_i; p_{i'j'k} + x_{i'j'k} + V_{ijk,i'j'k}; \\ tp_{hk} + xp_{hk} + V_{ijk,hk}) \qquad (4)$$

Using dioid notation, the above expression may be rewritten as:

$$\begin{aligned} x_{ijk} = \quad & t \oplus u_i \oplus p_{i'j'k} \otimes x_{i'j'k} \otimes V_{ijk,i'j'k} \\ & \oplus tp_{hk} \otimes xp_{hk} \otimes V_{ijk,hk} \end{aligned} \qquad (5)$$

If operation $j \in P$ is not the starting operation (i.e., has predecessors) on the job, then its processing start time x_{ijk} is determined by the maximum of either:

- The completion time of its direct predecessor, say $n \in P$, being processed on its correspondent machine, say $l \in M$.

- The completion of other operations ($j' \neq j$, and $j' \in P$), for other products i' that require processing on machine k. This is determined by the decision variables $V_{ijk,i'j'k}$ that determine which operation must be processed earlier on machine k.

- The completion of periodic maintenance activities PM_{hk} that require processing on machine M_k. This is determined by the decision variables $V_{ijk,hk}$ that generate the schedule between operation O_{ijk} and the periodic maintenance PM_{hk}.

This situation is formulated as follow:

$$x_{ijk} = max(\quad p_{inl} + x_{inl}; p_{i'j'k} + x_{i'j'k} + V_{ijk,i'j'k}; \\ tp_{hk} + xp_{hk} + V_{ijk,hk}) \qquad (6)$$

Using dioid notation, the above expression may be rewritten as:

$$\begin{aligned} x_{ijk} = \quad & p_{inl} \otimes x_{inl} \oplus p_{i'j'k} \otimes x_{i'j'k} \otimes V_{ijk,i'j'k} \\ & \oplus tp_{hk} \otimes xp_{hk} \otimes V_{ijk,hk} \end{aligned} \qquad (7)$$

In order to schedule the periodic maintenance activities and operations and maintenance tasks between each other that need processing on the same machine M_k, the following event-timing equation is added to the model:

$$xp_{hk} = max(\begin{array}{l} p_{ijk} + x_{ijk} + V_{hk,ijk}; \\ tp_{zk} + xp_{zk} + T_k + V_{hk,zk}) \end{array} \quad (8)$$

- In the last equation, the term "$p_{ijk} + x_{ijk} + V_{hk,ijk}$" represents the sequencing between operation O_{ijk} and periodic maintenance PM_{hk} via the decision variable $V_{hk,ijk}$.
- The term "$tp_{zk}+xp_{zk}+T_k+V_{hk,zk}$" represents the scheduling of maintenance operations that need processing on machine M_k between each other using the decision variable $V_{hk,zk}$, so that the time interval between two consecutive maintenance tasks is equal to the period T_k.

Using (max, +) algebra notation, equation 8 becomes:

$$xp_{hk} = \begin{array}{l} p_{ijk} \otimes x_{ijk} \otimes V_{hk,ijk} \\ \oplus tp_{zk} \otimes xp_{zk} \otimes T_k \otimes V_{hk,zk} \end{array} \quad (9)$$

To get feasible schedules, the different control variables in the model must be bounded and satisfy the following conditions:

$$V_{ijk,i'j'k} + V_{i'j'k,ijk} = B \quad (10)$$

$$max(V_{ijk,i'j'k}; V_{i'j'k,ijk}) = 0 \quad (11)$$

$$V_{ijk,hk} + V_{hk,ijk} = B \quad (12)$$

$$max(V_{ijk,hk}; V_{hk,ijk}) = 0 \quad (13)$$

$$V_{hk,zk} + V_{zk,hk} = B \quad (14)$$

$$max(V_{hk,zk}; V_{zk,hk}) = 0 \quad (15)$$

where B is chosen small enough (e.g., B is a large minus value).

The above equations force the decision variables to take either zero or a large minus value. In the proposed (max, +) model, decision variables (control variables) allow the controller to allocate machine time in a new planning horizon of production to processing products (jobs J_i) and maintenance tasks PM_{hk} at that machine. Values of control variables determine the order of jobs and maintenance activities at that machine. They are incorporated in the model as decision variables that serve as conflict resolvers in the sense that their values determine what value is attained by the system dynamic functions and consequently the actual starting times of the operations O_{ijk} and their respective jobs and maintenance tasks at their corresponding machines.

Using (max, +) notation, the above equations become:

$$V_{ijk,i'j'k} \otimes V_{i'j'k,ijk} = B \quad (16)$$

$$V_{ijk,i'j'k} \oplus V_{i'j'k,ijk} = 0 \quad (17)$$

$$V_{ijk,hk} \otimes V_{hk,ijk} = B \quad (18)$$

$$V_{ijk,hk} \oplus V_{hk,ijk} = 0 \quad (19)$$

$$V_{hk,zk} \otimes V_{zk,hk} = B \quad (20)$$

$$V_{hk,zk} \oplus V_{zk,hk} = 0 \quad (21)$$

where B is chosen small enough.

The period T_k is incorporated in the model, such that the duration between two consecutive periodic maintenance activities is equal to T_k. In addition, the first maintenance activity must start at date T_k. Consequently, the following (max, +) relations are introduced to the model:

$$max(-xp_{1k}, -xp_{2k}, ..., -xp_{xk}) = -T_k \quad (22)$$

The above equation means that the first repetitive periodic maintenance activity starts at date T_k.

$$max(\begin{array}{r} xp_{1k} + tp_{1k}; xp_{2k} + tp_{2k}; ...; \\ xp_{xk} + tp_{xk}) = \sum_{h=1}^{x} tp_{hk} + T_k * x \end{array} \quad (23)$$

The above equation means that the last repetitive periodic maintenance activity finishes at date $\sum_{h=1}^{x} tp_{hk} + T_k * x$.

Using (max, +) algebra notation, we have:

$$\bigoplus_{h=1}^{x} -xp_{hk} = -T_k \quad (24)$$

$$\begin{array}{c} \bigoplus_{h=1}^{x}(xp_{hk} \otimes tp_{hk}) = \\ \bigotimes_{h=1}^{x} tp_{hk} \underbrace{\otimes T_k \otimes T_k \otimes .. \otimes T_k}_{x\ times} \end{array} \quad (25)$$

where $1 \leqslant k \leqslant m$ and x is the number of repetitive periodic maintenance activities on machine M_k.

3.2 Max-Plus Scheduling Modeling Of HVLV Systems With Flexible Periodic Maintenance

This section deals with the flexible periodic maintenance case. In this situation, time intervals between two consecutive maintenance activities are not equals but they are known in advance. Indeed, the maintenance tasks are controllable.The jobs and the maintenance tasks are scheduled simultaneously, so that a regular criterion is optimized (Figure 2).

In this case, a (max, +) HVLV model with flexible periodic maintenance is proposed. Then $\forall \ 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant n_j, 1 \leqslant k \leqslant m$ and $1 \leqslant h \leqslant x$, two situations can be distinguished:

First, if operation $j \in P$ is the first (i.e., unprecedented) operation on the job, then its processing start time x_{ijk} is determined by the equation 4 and equation 5. Second, if operation $j \in P$ is not the starting operation (i.e., has predecessors) of the job, then its processing start time x_{ijk} is determined by the equation 6 and equation 7.

Now, in order to schedule the flexible periodic maintenance activities and operations and maintenance operations between each other, equation 8 is modified. In this equation, flexible periods (time intervals between two consecutive maintenance tasks) are not equals. Consequently, $\forall h$ and $z = 1...x, h \neq z$, the equation 8 becomes:

$$xp_{hk} = max(\quad p_{ijk} + x_{ijk} + V_{hk,ijk}; \\ tp_{zk} + xp_{zk} + \triangle_{hk,zk} + V_{hk,zk}) \quad (26)$$

Using (max, +) algebra notation, equation 26 becomes:

$$xp_{hk} = \quad p_{ijk} \otimes x_{ijk} \otimes V_{hk,ijk} \\ \oplus tp_{zk} \otimes xp_{zk} \otimes \triangle_{hk,zk} \otimes V_{hk,zk} \quad (27)$$

The maintenance activities can be considered as operations of particular jobs which will be processed on machines. Consequently, the developed event-timing-equations describing the dynamic of the system can be grouped into the following (max, +) matrix form:

$$X = T \oplus U \oplus C \otimes X \quad (28)$$

where:

- X is a (Nx1) (max, +) state vector (N is the total number of operations and maintenance tasks) which collects the starting times of operations and maintenance activities.
- T is a (Nx1) (max, +) vector which is composed of the beginning dates of the scheduling over the new planning horizon.
- U is a (Nx1) vector which contains the different dates at which the raw material of each product is fed to the system.
- C is a (NxN) appropriate (max, +) matrix describing the relationships among different state variables of the system. It contains the different decision variables.

Equation 26 shows that maintenance tasks are controllable. The jobs and the maintenance activities are scheduled simultaneously. Also maintenance operations are scheduled between them using decision variables. In order to incorporate the starting time δ_k of the first maintenance activity on each machine M_k in the model, we introduce the following equation:

$$max(-xp_{1k}, -xp_{2k}, ..., -xp_{xk}) = -\delta_k \quad (29)$$

Where δ_k is a constant.

As shown in Figure 2, time intervals between two consecutive flexible periodic maintenance tasks can be not equals. Indeed, we need to add the following constraint to the model in order to satisfy the duration between two consecutive maintenance operations:

$$\sum_{h=1}^{x} tp_{hk} + \delta_k - max(-\sum_{h,z=1;h\neq z}^{x} \triangle_{hk,zk}) \\ \leqslant max(xp_{1k} + tp_{1k}; xp_{2k} + tp_{2k}; ...; xp_{xk} + tp_{xk}) \\ \leqslant \sum_{h=1}^{x} tp_{hk} + \delta_k + max(\sum_{h,z=1;h\neq z}^{x} \triangle_{hk,zk}) \quad (30)$$

Note that the term $\sum_{h,z=1;h\neq z}^{x} \triangle_{hk,zk}$ in the inequality 30, is a $(x-1)$ by $(x-1)$ addition of the time intervals $\triangle_{hk,zk}$ (x is the number of flexible periodic maintenance on machine M_k). Also, all above constraints can be written using (max, +) algebra notation.

The above equation means that the completion time of the last flexible periodic maintenance activity is between $\sum_{h=1}^{x} tp_{hk} + \delta_k - max(-\sum_{h,z=1;h\neq z}^{x} \triangle_{hk,zk})$ and $\sum_{h=1}^{x} tp_{hk} + \delta_k + max(\sum_{h,z=1;h\neq z}^{x} \triangle_{hk,zk})$

4 ILLUSTRATIVE EXAMPLE

4.1 System Representation

For the sake of simplicity and without loss of generality, the application of the (max, +) model proposed in the Section 3 is explored below with an example of (6x6) Job-Shop system (6 products and 6 machines) (Figure 4).

This example is taken from an actual factory environment (Wang and Tang, 2011). It describes a HVLV system due to the wide variety of products (six kinds of products) and the processing times which are rela-

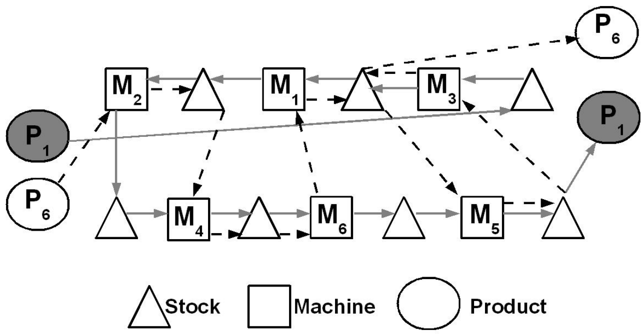

Figure 4: Job-shop HVLV system.

tively long. Data about routing and processing times for the 6 products are presented in Table 1.

Table 1: Production data.

job	Sequence (machine number, processing time)
J1	1(3,1) 2(1,3) 3(2,6) 4(4,7) 5(6,3) 6(5,6)
J2	1(2,8) 2(3,5) 3(5,10) 4(6,10) 5(1,10) 6(4,4)
J3	1(3,5) 2(4,4) 3(6,8) 4(1,9) 5(2,1) 6(5,7)
J4	1(2,5) 2(1,5) 3(3,5) 4(4,3) 5(5,8) 6(6,6)
J5	1(3,9) 2(2,3) 3(5,5) 4(6,4) 5(1,3) 6(4,1)
J6	1(2,3) 2(4,3) 3(6,9) 4(1,10) 5(5,4) 6(3,1)

In this example, the operations of six jobs are scheduled on six machines (Table 1) and we have 36 state variables x_{ijk} ($i = 1:6$, $j = 1:36$ and $k = 1:6$). In this section, PM is considered. Then, a non-linear optimization problem with constraints is applied to minimize the makespan.

Table 2: Repetitive periodic maintenance.

Machine	maintenance tasks
M_1	PM_{11} and PM_{21}
M_2	PM_{12}, PM_{22} and PM_{32}
M_5	PM_{15}, PM_{25} and PM_{35}
M_6	PM_{16}, PM_{26} and PM_{36}

We consider in the simulation example two periodic maintenance activities on M_1 and three periodic maintenance operations on M_2, M_5 and M_6 (Table 2).

Three flexible maintenance tasks are considered on M_3 and M_4 (Table 3).

Note that the allocated times to maintenance operations can be not equals on each machine (Table 4).

Table 5 shows the different values of periods on machines in the repetitive periodic maintenance case.

Table 6 shows the different values of time intervals between two consecutive maintenance tasks in the case of flexible periodic maintenance. These intervals are

Table 3: Flexible periodic maintenance.

Machine	maintenance tasks
M_3	PM_{13}, PM_{23} and PM_{33}
M_4	PM_{14}, PM_{24} and PM_{34}

Table 4: Duration of maintenance tasks.

Maintenance durations	Values
tp_{11}, tp_{23}, tp_{33}, tp_{36}	4
tp_{21}, tp_{14}, tp_{34}, tp_{16}, tp_{26}	3
$tp_{12}, tp_{15}, tp_{25}$	2
tp_{22}	5
tp_{32}	7
tp_{13}	1
tp_{24}, tp_{35}	6

different on machines M_3 and M_4. We have for example, $\triangle_{13,23} \neq \triangle_{13,33} \neq \triangle_{23,33}$.

4.2 Non-Linear Optimization Methodology

In this section, the proposed model is used to resolve the scheduling of the HVLV system with PM. Then, a non-linear optimization problem with constraints is applied. It deals with the minimization of the makespan and the total tardiness subject to JIT production. In the case of the makespan minimization, the control variables are only the decision variables (we suppose that $u_i = 0$). In the case of the total tardiness minimization, the control variables become u_i and the decision variables which generate feasible schedules.

4.2.1 Makespan Minimization

Let firstly define the makespan criterion of the HVLV system as follow:

$$C_{max} = max(C_i) = max(x_{iwk} + p_{iwk}) \tag{31}$$

where C_i is the completion time of product i and w is the last operation of product i.

Using the (max, +) notation, we have :

$$C_{max} = \bigoplus_{i=1}^{n} C_i = \bigoplus_{i=1}^{n} (x_{iwk} \otimes p_{iwk}) \tag{32}$$

where x_{iwk} is the starting time of the last operation w of product i on machine k and p_{iwk} is its corresponding processing time. Then, the non-linear optimization scheduling problem into (max, +) algebra is defined as:

$$C^*_{max} = minC_{max} = min(max(x_{iwk} + p_{iwk})) \tag{33}$$

Subject to the non-linear constraints (equations): (4), (6), (8), (10)-(15), (22), (23), (26), (29) and (30). B is chosen small enough.

Table 5: Periods of repetitive periodic maintenance.

Periods	Values
T_1	30
T_2	25
T_5	17
T_6	20

Table 6: Periods of flexible periodic maintenance.

Periods	Values
δ_3	15
δ_4	20
$\triangle_{13,23}$	7
$\triangle_{13,33}$	5
$\triangle_{23,33}$	6
$\triangle_{14,24}$	8
$\triangle_{14,34}$	9
$\triangle_{24,34}$	15

Applying the non-linear optimization problem to the example of (6x6) Job-Shop HVLV system shown in Section 4.1 with $t = u_i = 0$ for $i = 1 : 6$ and using the software LINGO as a tool for solving the optimization problem. Then, the obtained optimal value of the makespan $C^*_{max} = max(x_{165}+p_{165}; x_{264}+p_{264}; x_{365}+p_{365}; x_{466} + p_{466}; x_{564} + p_{564}; x_{663} + p_{663}) = 67$ time units. The corresponding schedules on the different machines based on the proposed (max, +) model are shown in Figure 5 that shows the order of each job Ji on each machine M_k.

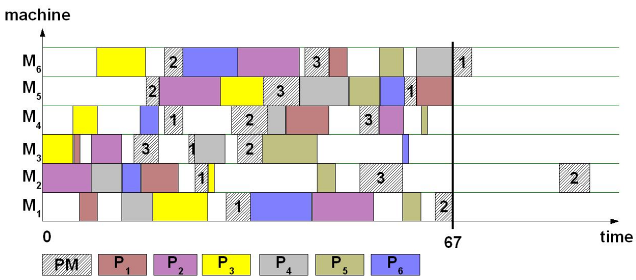

Figure 5: Operations scheduling on the machines.

The completion times C_i of the different products $i = 1 : 6$ are presented in Table 7.

The proposed model associated to a non-linear optimization algorithm in (max,+) algebra leads to an optimal value of the makespan $C^*_{max} = 67$ time units. A comparison between this result and the value of the optimal makespan in the static case (without PM) (Nasri et al., 2011), shows that while considering PM in the model increases the minimal value of the makespan (Figure 6). Moreover, The completion date of the maintenance activity PM_{22} is equal to 89. So, the starting date t of a new scheduling over a new planning horizon is equal to 89.

Table 7: Completion times of jobs.

Jobs	J1	J2	J3	J4	J5	J6
C_i	67	59	36	67	63	60

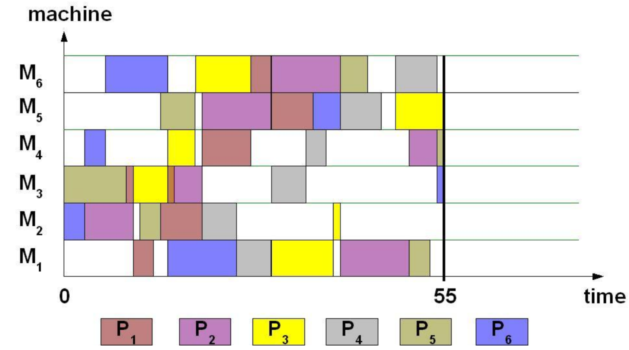

Figure 6: Operations scheduling on the machines

Figure 5 shows that the proposed (max, +) model is efficient and valid. The diagram of gantt shows that the periods between maintenance tasks are respected on each machine. Indeed, if we take the machine M_2 for example, it is clear that the time intervals between two consecutive maintenance tasks are equals to $T_2 = 25$. However, if we consider the machine M_3, time intervals are not equals, but the period between two consecutive maintenance operations is respected. So, the maintenance activity PM_{33} begins at $\delta_3 = 15$ and its completion date is equal to 19. The starting time of PM_{13} is equal to 24. So, the time interval between these two maintenance tasks is equal to $\triangle_{13,33} = 5$. Also, The completion date of the maintenance activity PM_{13} is equal to 25 and the starting time of PM_{23} is equal to 32. So, the time interval between these two maintenance tasks is equal to $\triangle_{13,23} = 7$. Moreover, the jobs and the maintenance operations are scheduled simultaneously and the maintenance tasks are scheduled between them, so that the makespan is minimal.

4.2.2 Total Tardiness Minimization: JIT Production

The objective of this section is to minimize the total tardiness criterion for a non-linear optimization using the (max, +) algebra and subject to JIT production (Figure 7)

As far as we know, there are few researches about scheduling problems in the literature that deal with the total tardiness minimization. Moreover, all these researches don't handle the JIT production criterion in the scheduling problems. In this section, the total tardiness is minimized, so that the JIT production is satisfied.

Let now define the following criterion:

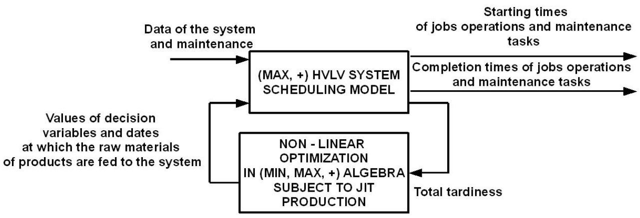

Figure 7: Full scheme of dynamic total tardiness optimization with maintenance.

$$T = T_d + T_e \tag{34}$$

Using the (max, +) notation, equation 34 becomes:

$$T = T_d \otimes T_e \tag{35}$$

where $T_d = \sum_{i=1}^{n} T_i$, and $T_e = -\sum_{i=1}^{n} u_i$.

Using the (max, +) notation, we have:

$T_d = \bigotimes_{i=1}^{n} T_i$ and $T_e = -\bigotimes_{i=1}^{n} u_i$

Let define the total tardiness criterion $T_i = max(x_{iwk} + p_{iwk} - D_i; 0)$, for $i = 1 : n$, and $k = 1 : m$ (n is the number of products).

where w is the last operation of product i and x_{iwk} is the starting time of the last operation w of product i on machine k and p_{iwk} is its corresponding processing time. D_i is the due date of the product i.

- T_d reflects the due date tracking error.
- T_e reflects the control effort (JIT criterion). The minimization of T_e would lead to maximization of the dates u_i at which the raw material of each product is fed to the system as late as possible. Consequently, the starting time of the first operation x_{i1k} of each product i will be equal to u_i.

Then, the non-linear optimization scheduling problem into (max, +) algebra is defined as follow:

$$T^* = minT = min(T_d + T_e) \tag{36}$$

Subject to the non-linear constraints (equations): (4), (6), (8), (10)-(15), (22), (23), (26), (29) and (30). B is chosen small enough. We have also the following inequalities:

$$C_i = x_{iwk} + p_{iwk} \leqslant D_i \tag{37}$$

$$x_{i1k} \geqslant u_i \tag{38}$$

Where C_i is the completion time of product i and w is the last operation of product i.

The above non-linear optimization problem is applied to the example of (6x6) Job-Shop HVLV system shown in Section 4.1 with $t = 0$ and for the different chosen due dates D_i (Table 8). The software LINGO is used as a tool for solving the optimization problem. Then, the obtained optimal value $T^* = -110$ time units. This optimal criterion corresponds to the following completion times of products (Table 8):

Table 8: Due dates and completion times of jobs.

Jobs	J1	J2	J3	J4	J5	J6
D_i	73	67	70	69	74	76
C_i	73	67	40	60	71	66

The corresponding scheduling on the different machines based on the proposed (max, +) model is represented by the following diagram of Gantt (Figure 8):

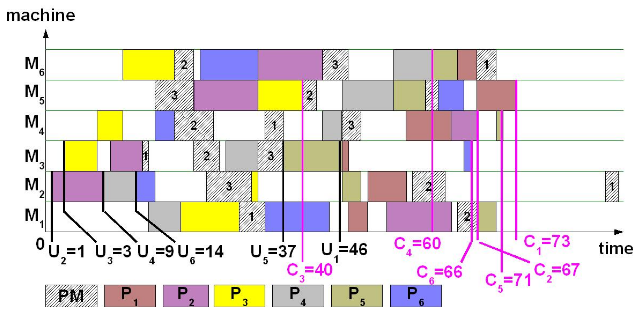

Figure 8: Operations scheduling on the machines.

Table 8 shows that the due dates D_i ($i = 1 : 6$) of the different products are met. Then for all $i = 1 : 6$, $C_i \leqslant D_i$. Moreover, Table 9 shows that the JIT production criterion is satisfied. Indeed, the starting times of each first operation of the product i, x_{i1k}, is equal to the date at which the raw material of each product i, u_i, is fed to the system. Then, the proposed criterion Te allows the maximization of the input time u_i, so that the raw material of products is fed to the system as late as possible. As a consequence, the internal buffer levels are kept as low as possible. Moreover, The completion date of the maintenance activity PM_{12} is equal to 88. So, the starting date t of a new scheduling over a new planning horizon is equal to 88.

Table 9: Controlled dates of the first operation of the products.

Jobs	J1	J2	J3	J4	J5	J6
u_i	46	1	3	9	37	14
x_{i1k}	46	1	3	9	37	14

5 CONCLUSION

The objective of this work is to build a (max, +) algebraic model for scheduling, optimization, and control of HVLV systems while periodic preventive maintenance is considered. Two situations concerning maintenance are investigated simultaneously in this paper. In the first one, maintenance tasks are periodically fixed: maintenance is required after a periodic time interval (all periods are equals on each machine). In the second one, time intervals between two consecutive maintenance activities are not equals (flexible periodic maintenance). The jobs and both situations of maintenance operations are scheduled simultaneously. Moreover, the maintenance tasks are scheduled between them, so that a regular criterion is optimized. A non-linear optimization problem with constraints is then solved into (max, +) algebra to minimize two criteria. The total tardiness criterion is extended to solve a JIT production problem. The simulation results show that the proposed model can be a good tool for the control and optimization of HVLV systems with maintenance.

In real-world applications for HVLV systems, various uncertainty aspects of the system will perturb its behavior (processing times, set-up times, etc). In this context, next research work will be done to improve the proposed model to make it robust in presence of perturbations, such that it can deal with changeover in HVLV systems. Also, analytical techniques for differentiation and optimization in (max, +) algebra should be developed to effectively use dioid algebraic models in solving scheduling problems.

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